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SUMMARY OF A STRICTLY POSITIVE REAL SYSTEM FOR CONTROLLING A WHEELCHAIR MODEL USING A PARAMETER- DEPENDENT LYAPUNOV FUNCTION

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Abstract: The objective of this undergraduate research project was to synthesize the controller for a breath- and suction-controlled wheelchair model, described in Mazo et al. (1995), using the Strictly Positive Real (ERP) method and considering polytopic uncertainties in the system plant with random noise at the input, via Linear Matrix Inequalities (LMIs) and a Lyapunov function $V(x) = x^T P(\alpha) x$, where the matrix $P(\alpha)$ depends on the polytopic uncertainties present in the model. To carry out the proposed work, Matlab software was used both to perform the calculations necessary for the synthesis of the ERP system and to conduct simulations of the synthesized system, applied to the wheelchair model. The results obtained were satisfactory, achieving a strictly positive-definite system of the plant model through the LMIs and also good results in the system simulations regarding a sinusoidal input, presenting short stabilization times of the plant parameters with respect to time. Throughout this work, it was possible to gain extensive knowledge of advanced and modern control techniques, widely used in the context of contemporary control engineering, which have proven to be highly effective and capable of stabilizing systems with a high degree of complexity.

Keywords: ERP System. Lyapunov Function. Wheelchair. Polytopic Uncertainties. Linear Matrix Inequalities (LMIs).

INTRODUCTION

The demand for technological advances in the healthcare sector continues to grow; with this in mind, the concept of a wheelchair incorporating control technologies fits perfectly into the current land-

scape. More specifically, the breath- and suction-controlled wheelchair, studied and mathematically modeled in Mazo et al. (1995) and Gaino et al. (2010; 2013), aims to meet the needs—often neglected by wheelchairs that rely on manual hand operation for locomotion—of patients who lack hand movement.

Furthermore, this work aims to provide a deeper understanding of control systems theory, which is becoming increasingly popular and evolving at an unprecedented rate, utilizing advanced and modern techniques. More specifically, this work aimed to explore and develop a Strictly Positive Real System (ERP), extensively described and studied in Covacic (2006), which is a class of control systems belonging to the category of robust control, where techniques are developed to ensure the stability of a system even in the presence of uncertainties and unknown disturbances in its plant. To ensure the development of this system, methods involving Linear Matrix Inequalities (LMIs) and a Lyapunov Function, discussed in Covacic (2006), were used.

In the Journal Portal database, using the keywords “wheelchair” and “LMI,” the most recent publication found was the article by Feng et al. (2021). Our work differs from theirs in that it analyzes robustness by considering parameter variation, a condition not addressed in Feng et al. (2021).

Strictly Real Positive (ERP) Systems

ERP systems are passive, asymptotically stable systems whose transfer function zeros have a negative real part. Furthermore, the negative feedback in this type of system is internally stable (Covacic et al., 2010; Covacic & Gaino, 2014).

The importance of ERP systems in the analysis and design of control systems is demonstrated by the results obtained regarding the stability of these systems, such as Popov's asymptotic hyperstability (Anderson, 1968). These results have a wide range of applications, such as in the design of control systems, in control systems with variable structure, and in the stabilization of uncertain systems with output feedback (Covacic et al., 2010).

Consider a time-invariant linear plant $G_p(s)$ represented by:

$$\begin{aligned} \dot{x} &= A_p x + B_p u, \\ y &= C_p x, \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^{(n)}$ is the state vector, $u \in \mathbb{R}^{(m)}$ is the control input, $y \in \mathbb{R}^{(m)}$ is the system output, $A_p \in \mathbb{R}^{(n \times n)}$ is the system characteristic matrix, $B_p \in \mathbb{R}^{m \times n}$ the input matrix of the system, and $C_p \in \mathbb{R}^{m \times n}$ the output matrix of the system (Aguirre, 2007).

ERP systems were defined in (Anderson, 1968). The necessary and sufficient condition for an ERP system is given in Theorem 1 (Anderson, 1968):

Theorem 1 (Anderson, 1968) The transfer matrix of the plant, given by $G_p(s) = C_p (sI - A_p)^{-1} B_p$, is ERP if and only if there exists a matrix $P = P^T$, such that:

$$\begin{aligned} PA_p + A_p^T P &< 0, \\ B_p^T P &= C_p, \end{aligned} \quad (2)$$

$$P > 0.$$

Next, necessary and sufficient conditions are established for the existence of a matrix K that makes the system in Figure 1, with input $V(s)$ and output $Y(s)$, ERP.



Figure 1 – System with feedback K .

Theorem 2 (Teixeira, 1989; Kaufman et al., 1994) Consider Figure 1. There exists a constant matrix K such that the system in Figure 1, with input $V(s)$ and output $Y(s)$, is ERP if and only if the following conditions are satisfied:

1. $C_p B_p = (C_p B_p)^T > 0$;
2. All transfer zeros of the plant $\{A_p, B_p, C_p\}$ have a negative real part.

Lyapunov Function

In classical mechanics, it is known that the stability of a vibrating system occurs when its total energy continuously decreases to a certain equilibrium point, and the Lyapunov function is a generalization of this theory. Developed by the Russian mathematician Aleksandr Lyapunov, the Lyapunov function is a mathematical function that represents the “energy” contained in a given system and allows one to verify its stability. In this work, the function of interest will be represented as:

$$V(x) = x^T P_0(\alpha)x, \quad (3)$$

where x is the vector of the system's state variables and $P_0(\alpha)$ is the parameter-dependent Lyapunov matrix, with α given in (6).

If the Lyapunov function is positive definite and its derivative with respect to time is negative definite for every

time instant greater than zero—that is, if the system’s energy always dissipates over time—then the system can be said to be asymptotically stable. This statement can be verified in Theorem 3 and Definitions 1, 2, 3, 4, and 5, given in Slotine; Li (1991).

Definition 1 (Slotine; Li, 1991) A continuous scalar function $V(x)$ is said to be locally positive-definite if $V(0) = 0$ and, in a ball B_{r_0} , for all $x \neq 0$, $V(x) > 0$. If $V(0) = 0$ and the pre e property holds for the entire state space, then $V(x)$ is said to be globally positive-definite.

Definition 2 (Slotine; Li, 1991) A function $V(x)$ is negative-definite if $-V(x)$ is positive-definite.

Definition 3 (Slotine; Li, 1991) A function $V(x)$ is positive semidefinite if $V(0) = 0$ and $V(x) \geq 0$ for $x \neq 0$.

Definition 4 (Slotine; Li, 1991) A function $V(x)$ is negative semidefinite if $-V(x)$ is positive semidefinite.

Theorem 3 (Slotine; Li, 1991): If in a ball B_{r_0} , there exists a scalar function $V(x)$ with continuous first-order partial derivatives, such that:

- $V(x)$ is positive definite (locally in B_{r_0}),
- it is negative semidefinite (locally in B_{r_0}),

then the equilibrium point is stable. If, furthermore, the derivative is locally negative and defined on B_{r_0} , then the stability is asymptotic.

Definition 5 (Slotine; Li, 1991) If, in a ball B_{r_0} , the function $V(x)$ is positive definite, has continuous partial derivatives, and its time derivative along any trajectory in the system is positive semidefinite, i.e.:

$$\dot{V}(x) \leq 0, \quad (4)$$

as shown in Figure 2, then $V(x)$ is a Lyapunov function of the system.

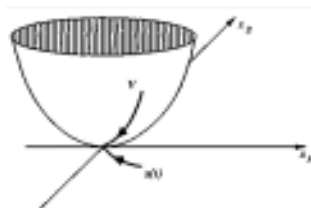


Figure 2 – Illustration of Definition 5 (Slotine; Li, 1991).

Polytopic Uncertainties

In practical situations, there are generally uncertainties in the plant parameters. These uncertainties must be considered in the design of control systems. The conditions proposed in this work apply to plants similar to $G_p(s)$ in (1), with uncertainties in the characteristic matrix A and the input matrix B . Consider, then, the plant $G_p(s)$ whose state-space representation is described by (Covacic; Gaino, 2014):

$$\begin{aligned} \dot{x} &= A_p(\alpha)x + B_p(\alpha)u, \\ y &= C_p x, \end{aligned} \quad (5)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ and $\alpha \in \mathbb{R}^r$ given by:

$$\alpha = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_{r-1} \ \alpha_r]^T, \quad (6)$$

where $\alpha_i \geq 0$, $i = 1, 2, \dots, r - 1$, r are unknown variables that vary over time at a bounded rate of change, such that $\alpha_1 + \alpha_2 + \dots + \alpha_{(r-1)} + \alpha_r = 1$ (Covacic; Gaino, 2014).

The rates of change of α_i , $i = 1, 2, \dots, r - 1, r$, are bounded, that is:

$$|\alpha_k| \leq \varphi_k < \infty, k = 1, \dots, r. \quad (7)$$

Polytopic uncertainties can be understood as a margin of error associated with the elements of the matrices A_p and B_p , which have all their parameters known. The matrices $A_p(\alpha)$ and $B_p(\alpha)$, which have unknown parameters, can be described as:

$$A_p(\alpha) = \sum_{i=1}^r \alpha_i A_{pi}$$

$$B_p(\alpha) = \sum_{i=1}^r \alpha_i B_{pi}, \quad (8)$$

where A_{pi} and B_{pi} , $i = 1, \dots, r$ are known and constant matrices.

Also in (Covacic, Gaino, 2014), the development of an ERP system with polytopic uncertainties in the plant is proposed through the design of a control law of the form:

$$u(t) = -K \cdot y(t), \quad (9)$$

and a matrix F in series with the system output, as illustrated in Figure 3:

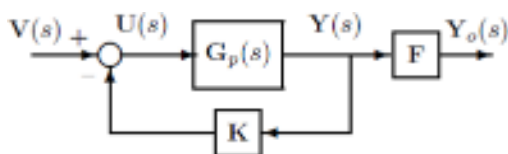


Figure 3 – ERP system (Covacic, Gaino, 2014).

For this specific situation, the ERP system is obtained from the following theorem:

Theorem 4 (Covacic, Gaino, 2014): Consider the plant (5) with control law (9). A sufficient condition for the existence of matrices F and K that make the system illustrated in Figure 3 ERP is the existence of matrices $P_{0i} = P_{0i}^T$, R, and F that satisfy the following LMIs:

$$\frac{1}{2}(A_{pi}^T \cdot P_{0j} + P_{0j} \cdot A_{pi} + A_{pj}^T \cdot P_{0i} + P_{0i} \cdot A_{pj}) - C_p^T$$

$$\cdot R \cdot C_p - C_p^T \cdot R^T \cdot C_p + \sum_{i=1}^r \varphi_i P_{0i} < 0, \quad (10)$$

$$P_{0i} \cdot B_{pi} = C^T \cdot F^T, \quad (11)$$

$$P_{0i} > 0, \quad (12)$$

for $i, j = 1, \dots, r$. When the above conditions are satisfied, the matrix K is given by:

$$K = (F^T)^{-1} \cdot R. \quad (13)$$

Wheelchair Model

The project utilized a model of a breath- and suction-controlled wheelchair in the field of assistive technology. A breath- and suction-controlled wheelchair is of great importance for patients who lack hand movement and, consequently, would be unable to control the wheelchair manually using a joystick. The mathematical model of the wheelchair used was based on the model presented in Mazo et al. (1995), as shown in Figure 4, and its state-space modeling presented in Gaino et al. (2010;2013).

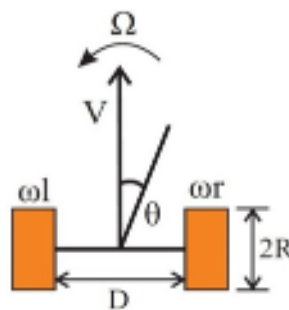


Figure 4 – Wheelchair model in Mazo et al. (1995).

In the model shown in Figure 4, we have the following definitions:

- R is the radius of the rear wheels;
- D is the distance between the rear wheels;
- ω_l is the angular frequency of the left wheel;

- ω_r is the angular frequency of the right wheel;
- V is the linear velocity of the chair;
- Ω is the angular velocity of the chair;
- θ is the angle of rotation of the chair.

According to the state-space modeling of the chair, which already includes a Proportional-Integral-Derivative (PID) controller in its composition, as described in Gaino et al. (2010;2013), the plant matrices of the model are:

$$A = \left[a_{11} \ 0 \ \frac{K.m}{T} \ 0 \ 0 \ a_{11} \ 0 \ \frac{K.m}{T} \ a_{22} \ 0 \ -\frac{I}{K_2} \ 0 \ 0 \ a_{22} \ 0 \ -\frac{I}{K_2} \right], \quad (14)$$

$$a_{11} = \frac{(-K_2 \cdot (K_1 + 1) \cdot K \cdot m - 1) \cdot K_2}{T}, \quad (15)$$

$$a_{22} = -I \cdot (K_1 + 1), \quad (16)$$

$$B = \left[\begin{array}{cccc} \frac{K_2 \cdot K_1 \cdot K \cdot m}{R \cdot T} & \frac{K_2 \cdot K_1 \cdot K \cdot m \cdot D}{2 \cdot R \cdot T} & \frac{K_2 \cdot K_1 \cdot K \cdot m}{R \cdot T} & -\frac{K_2 \cdot K_1 \cdot K \cdot m \cdot D}{2 \cdot R \cdot T} \\ 0 & 0 & 0 & 0 \end{array} \right], \quad (17)$$

$$C = [1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0], \quad (18)$$

where:

- K is the transfer function gain of the chair's motors;
- K_1 is the proportional constant of the proportional controllers;
- K_2 is the proportional constant of the Proportional-Integral (PI) controllers;
- m is the gear ratio;
- I is the integral constant of PI controllers;
- T is the motor time constant.

METHODOLOGY AND EXPERIMENTAL PROCEDURES

Considering the matrices in (14) and (17), the model in Figure 4, and the variations in the plant's polytopic uncertainties, the following practical values were adopted for the parameters, as shown in Table 1 below:

$K_{min} = 21,6/\pi$	$R = 0,15$ [m]
$K_{max} = 26,4/\pi$	$m = 0,1$
$K_1 = 0,1768$	$T_{min} = 0,117$ [s]
$K_2 = 0,18534$	$T_{max} = 0,143$ [s]
$I = 4,8753$	$D = 0,7$ [m]

Table 1 – Experimental parameter values.

Source: (The author, adapted from the nominal values given in Gaino et al. (2010; 2013).)

Taking into account the different combinations of maximum and minimum parameters present in the matrices, due to the presence of polytopic uncertainties, four different matrices A and matrices B were obtained with the stipulated experimental values, each with the following values:

$$A_1 = [16,4759 \ 0 \ 5,8765 \ 0 \ 0 \ 16,4759 \ 0 \ 5,8765 \ -79,4875 \ 0 \ -26,3046 \ 0 \ 0 \ -79,4875 \ 0 \ -26,3046], \quad (19)$$

$$A_2 = [18,2629 \ 0 \ 4,8080 \ 0 \ 0 \ 18,2629 \ 0 \ 4,8080 \ 105,6533 \ 0 \ -26,3046 \ 0 \ 0 \ -105,6533 \ 0 \ -26,3046] \quad (20)$$

$$A_3 = [16,1910 \ 0 \ 7,1824 \ 0 \ 0 \ 16,1910 \ 0 \ 7,1824 \ -65,0352 \ 0 \ -26,3046 \ 0 \ 0 \ -65,0352 \ 0 \ -26,3046] \quad (21)$$

$$A_4 = [18,0299 \ 0 \ 5,8765 \ 0 \ 0 \ 18,0299 \ 0 \ 5,8765 \ -86,4436 \ 0 \ -26,3046 \ 0 \ 0 \ -86,4436 \ 0 \ -26,3046], \quad (22)$$

$$B_1 = [1, 2837 \ 0, 4493 \ 1, 2837 \ -0, 4493 \ 0 \ 0 \ 0 \ 0], \quad (23)$$

$$B_2 = [1, 0503 \ 0, 3676 \ 1, 0503 \ -0, 3676 \ 0 \ 0 \ 0 \ 0], \quad (24)$$

$$B_3 = [1, 5690 \ 0, 5491 \ 1, 5690 \ -0, 5491 \ 0 \ 0 \ 0 \ 0], \quad (25)$$

$$B_4 = [1, 2837 \ 0, 4493 \ 1, 2837 \ -0, 4493 \ 0 \ 0 \ 0 \ 0], \quad (26)$$

The LMIs used in this work were derived from the combinations of these matrices and arranged in the Matlab software console according to (10), (11), and (12).

In inequality (10), we observe that the value of φ_i is bounded according to (7), and the value adopted in this study for this variable was $\varphi_i = 0$. In Covacic; Gaino (2014), inequality (7) guarantees the following inequality:

$$\sum_{i=1}^r \alpha_i P_{0i} \geq \sum_{i=1}^r \alpha_i P_{0i}, \quad (27)$$

Since the variation of the polytopic uncertainties in the system with respect to time is zero, that is, $\dot{x}_i = 0$, we obtain:

$$\sum_{i=1}^r \alpha_i P_{0i} = \sum_{i=1}^r \alpha_i P_{0i}, \quad (28)$$

therefore, according to (27) and (28), even by adopting the value $\varphi_i = 0$, we can ensure that the system synthesized from (10), (11), and (12) is ERP.

Note 1 To perform the calculation of equation (11), it was necessary to use an approximation technique for an LMI, since it is not possible to solve Linear Matrix Equalities (LMEs) in MATLAB.

The approximation used in equation (11) was:

$$\|P_i \cdot B_{pi} - C_p^T \cdot F^T\| < \lambda \quad (29)$$

where the value of λ must be sufficiently close to zero to allow for the solution of (11). The value used in this work was $\lambda = 10^{-5}$.

The approximation in (29) corresponds to:

$$(P_{0i} \cdot B_p - C^T \cdot F^T)^T \cdot (P_{0i} \cdot B_p - C^T \cdot F^T) < \lambda^2 \cdot I, \quad (30)$$

that is:

$$\lambda^2 \cdot I - (P \cdot B - C^T \cdot F^T)^T \cdot (P \cdot B - C^T \cdot F^T) > 0, \quad (31)$$

Thus, inequality (31), according to mathematical manipulations involving the Schur complement, is equivalent to the following LMI:

$$\lambda^2 I (P_i B_{pi} - C_p^T F^T)^T P_i B_{pi} - C_p^T F^T I > 0, \quad (32)$$

which can be solved using the tools available in MATLAB.

RESULTS

The matrices K, F, and R, obtained via the LMIs (10), (12), and (32), were:

$$K = [12,7990 \ 12,7990 \ 36,5352 \ -36,5352], \quad (33)$$

$$F = [0,3774 \ 0,3774 \ 0,1322 \ -0,1322], \quad (34)$$

$$R = [9,6609 \ 0 \ 0 \ 9,6609], \quad (35)$$

Four matrices, P_1 , P_2 , $P_{(3)}$, and P_4 , were also obtained due to polytopic uncertainties:

$$P_1 = [0,2941 \ -0,0001 \ 0 \ 0 \ -0,0001 \ 0,2941 \ 0 \ 0 \ 0 \ 0,1403 \ 0 \ 0 \ 0 \ 0,1403], \quad (36)$$

$$P_2 = [0,3595 \ -0,0002 \ 0 \ 0 \ -0,0002 \ 0,3595 \ 0 \ 0 \ 0 \ 0,0955 \ 0 \ 0 \ 0 \ 0,0955], \quad (37)$$

$$P_3 = [0,2406 \ -0,0001 \ 0 \ 0 \ -0,0001 \ 0,2406 \ 0 \ 0 \ 0 \ 0,1880 \ 0 \ 0 \ 0 \ 0,1880], \quad (38)$$

$$P_4 = [0,2941 \ -0,0001 \ 0 \ 0 \ -0,0001 \ 0,2941 \ 0 \ 0 \ 0 \ 0,1534 \ 0 \ 0 \ 0 \ 0,1534], \quad (39)$$

Checking the conditions of Theorem 2, we obtain the following values for the first

condition $FC_p B_p = (FC_p B_p)^T > 0$, for each matrix B_p :

$$F \cdot C_p \cdot B_{p1} = [0,9690 \ 0,0000 \ 0,0000 \ 0,1188], (40)$$

$$F \cdot C_p \cdot B_{p2} = [0,7928 \ 0,0000 \ 0,0000 \ 0,0972], (41)$$

$$F \cdot C_p \cdot B_{p3} = [1,1843 \ 0,0000 \ 0,0000 \ 0,1452], (42)$$

$$F \cdot C_p \cdot B_{p4} = [0,9690 \ 0,0000 \ 0,0000 \ 0,1188], (43)$$

Next, the following values were obtained for all possible combinations of the second condition, regarding the transmission zeros of $(A_\pi - B_\pi \cdot K \cdot C_p)$, with the same result for all vertices of the polytope:

$$z = [-26,3046 \ -26,3046], (44)$$

Thus, it can be observed that the conditions of Theorem 2 were satisfied and the system obtained in this work is ERP.

In Covacic (2006), for the following uncertain system:

$$\begin{aligned} \dot{x} &= A(\alpha)x + B(\alpha)(u + \zeta(t, x)), \\ y &= C(\alpha)x, \end{aligned} (45)$$

the following assumptions, relevant to the present work, are made:

1. Let $|\zeta(t, x)| = \|\zeta(t, x)\|_1 = |\zeta_1(t, x)| + \dots + |\zeta_m(t, x)|$, where $\zeta(t, x)^T = [\zeta_1(t, x), \dots, \zeta_m(t, x)]$, then there exist positive constants a and b such that $|\zeta(t, x)| \leq a|x| + b$;

2. The matrix $B(\alpha) = B + \Delta B$, where $\Delta B = B \Delta \dot{B}$. There exist and are known l_i , $i = 1, \dots, m$, such that $1 + \dot{B}_{\Delta i, i} - |\Delta \dot{B}_{i, 1}| - \dots - |\Delta \dot{B}_{i, i-1}| - |\Delta \dot{B}_{i, i+1}| - \dots - |\dot{B}_{i, m}| > l_i > 0, \forall i \in [1, \dots, m]$, and l is defined as $l = \min[l_1, l_2, \dots, l_m]$.

Simulations were performed based on the system described in (45) in response to the input in (46), using the matrices A , B , and C of the polytope's vertices, the matrices

K and F , previously calculated, and a control law given by:

$$u = -K_0 y - \beta \text{sign}(F_y) + \zeta(t, x), (46)$$

where $\zeta(t, x)$ is a random noise at the input with amplitude between -2 and 2 , and β is defined by:

$$\beta > \frac{|\Delta \dot{B}| \cdot |K_0 y| + |l + \Delta \dot{B}|(a|x| + b)}{l}. (47)$$

Since the noise was applied only at the system input, and there are no uncertainties in matrix B , then $\Delta = 0$, with $a = 2$, and the value of $\beta = 7$ used is appropriate, according to (47).

The results of the simulations, with initial state $x_0 = [0.25 \ 0.25 \ 0 \ 0]^T$, can be seen below, where Figures 5 and 6 show the angular velocity response plots for the left and right wheels, respectively, relative to the input, and Figures 7 and 8 show the angular position plots for the left and right wheels, respectively, relative to the input (46) used:

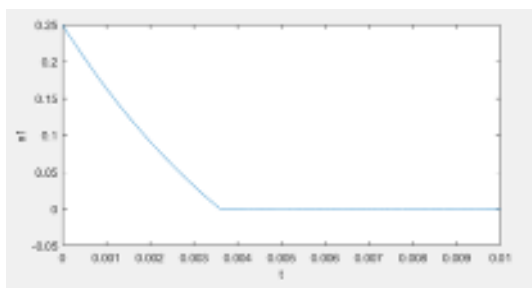


Figure 5 – Angular velocity of the left wheel ω_1 .

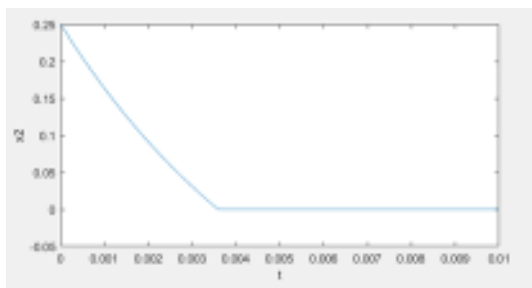


Figure 6 – Angular velocity of the right wheel ω_2 .

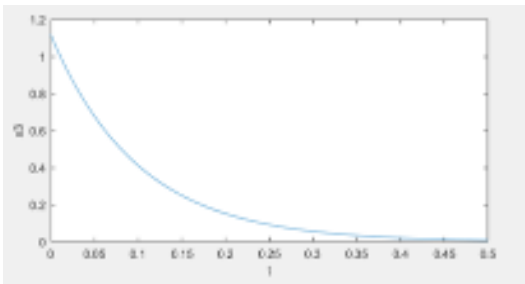


Figure 7 – Angular position of the left wheel.

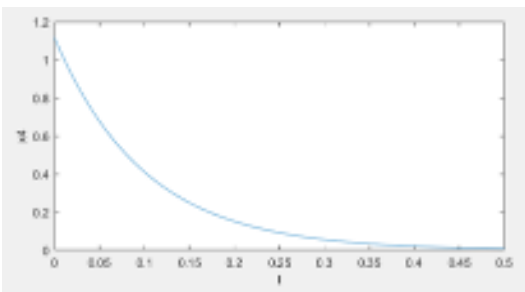


Figure 8 – Angular position of the right wheel.

The results obtained were satisfactory, consistent with an ERP system, showing good stabilization times for both wheels of the wheelchair in response to the system input.

CONCLUSION

We successfully achieved the initial proposal presented in the introduction of this study, despite all the complications and problems that arose along the way, involving doubts regarding the theoretical understanding of the work and necessary adjustments and corrections throughout the practical phase. First, the study conducted in the first stage of the work, aimed at establishing a solid theoretical foundation to subsequently carry out the practical phase with proficiency, was based on articles and studies provided by the supervising professor as well as sources found in the literature, providing an excellent foundation for the subsequent sta-

ges of the project and fulfilling the primary objective of studying modern control techniques in greater depth. Next, the practical part proved to be very challenging, as well as fascinating and intriguing, consolidating the theoretical knowledge previously obtained, with new challenges and problems arising at every turn, increasingly raising the level of effort and knowledge invested in the project. Regarding the results obtained in the final part of the work, it was possible to fully fulfill the objective of developing an ERP system for the wheelchair model in question, presenting mathematical data that corroborate the practical results obtained in the simulations, as well as confirming the success of the work through graphs of the system's response over time, presenting sufficient data consistent with an ERP system.

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