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EXERCÍCIOS RESOLVIDOS SOBRE MÁQUINAS ELÉTRICAS DE CA SOLVED EXERCISES ON AC MACHINES

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Abstract: In this chapter, we present the process for solving exercises on synchronous motors, synchronous generators, and single-phase and three-phase induction motors. The objective is to provide students with a comprehensive guide for studying and understanding the mathematics applied to alternating-current electric machines.

Keywords: Synchronous machines. Induction motor. Power factor improvement. Exercises.

REVIEW OF AC MACHINES THEORY

To get a rotating magnetic field, we can connect three coils in a star configuration, every 120 degrees out of phase, and apply three alternating current voltage waves, also 120 degrees out of phase. The interaction of the three alternating current waves in the coils creates a rotating magnetic field. The speed of this magnetic field is called synchronous speed.

In a three-phase synchronous generator, there are rotating parts that make up the rotor or shaft, and stationary parts that make up the stator. The shaft is moving by hydraulic or steam turbines at the synchronous speed of the rotating magnetic field. This speed, measured in revolutions per minute (rpm), is given by the formula $N_s = 120f / p$, where f represents the frequency of the generated voltage in Hertz and p is the number of poles of the machine.

The three-phase synchronous generator consists of a field coil powered by direct current and a three-phase armature winding comprising three coils, which generate three alternating current voltage waves. When the synchronous generator has a rotating field coil, the three-phase armature winding is located in the stator, and vice versa.

To operate a three-phase synchronous generator with a rotating armature winding, the stator field coil is energized with direct current voltage to create a stationary magnetic field. Then, mechanical force is used to spin the rotor's three-phase armature winding within this static magnetic field. As a result, three induced alternating-voltage waves are generated in the armature's three-phase winding, which are transmitted to the load via slip rings and brushes. To operate a synchronous generator with a rotating field coil, the rotor's field coil is excited by direct current, which creates a magnetic field when a current flows through it. Then, mechanical force is applied to rotate this coil and its magnetic field, which induces three sets of alternating current voltage waves in the stator's three-phase armature winding.

The synchronous generator is crucial for producing alternating current, which is essential for the daily functioning of an industrialized country. This type of generator is found in hydroelectric power plants and is responsible for generating and supplying most of the electric power used.

The three-phase synchronous motor operates like a three-phase synchronous generator but in reverse as a motor. To create a stationary magnetic field, a direct current voltage is applied to the field coil. Meanwhile, three alternating current voltage waves are applied to the three-phase winding to generate a rotating magnetic field. This field rotates at synchronous speed and produces a torque that causes the shaft to rotate at the same speed. The poles of the rotating magnetic field attract the opposite poles of the stationary magnetic field, resulting in automatic shaft rotation.

The synchronous motor is used in electric clocks, time-recording devices, frequency converters, and power-factor improvement.

In an induction motor, the stator winding is supplied with alternating current. The rotor, whether squirrel-cage or wound, is not connected to an external power source. Instead, the rotor is powered by magnetic induction and never reaches the speed of the rotating magnetic field.

The three-phase induction motor operates as follows: stator coils are supplied with alternating current, generating a rotating magnetic field that induces voltages in the rotor, thereby generating induced currents. These induced currents create a magnetic field in the rotor. The magnetic fields of the stator and rotor tend to align, resulting in torque. The rotating magnetic field causes the machine's rotor to rotate. The difference between the speed of the rotating magnetic field (N_s) and the rotor speed (N) is called slip, defined as $s\% = 100\ (N_s - N)\ /\ N_s$, where $N_s = 120f\,/\,p$.

In locations where a three-phase power supply is not available, single-phase induction motors are used. These motors operate with only one power supply phase, meaning they don't produce a rotating magnetic field and can't start independently. To address this issue, several types of single-phase induction motors have been developed: split-phase, capacitor-start, permanent-split capacitor, two-capacitor, and shaded-pole.

EXAMPLE 1

Consider a three-phase synchronous motor with the following specifications: voltage levels of 220 V, 380 V, 440 V, and 760 V; active power of 0.4 kW; power factor of 0.8; frequency of 60 Hz; speed of 1800 rpm. Your task is to determine the motor's absorbed current at each voltage level, the apparent power, reactive power, and the number of poles.

Solution

For a three-phase alternating current circuit, the total three-phase active power is given by:

$$P = \sqrt{3} V_L I_L \cos\phi \quad \rightarrow \quad I_L = P / \left(\sqrt{3} V_L \cos\phi\right)$$

For an input voltage of 220 V, the motor requires a current of:

$$I_L = 400 / \left(\sqrt{3}\, 220 \times 0.8\right) = 1.31 \quad A$$

For the remaining voltage levels, we have the following:

$$I_L = 400 / \left(\sqrt{3}\, 380 \times 0.8\right) = 0.76 \quad A$$

$$I_L = 400 / \left(\sqrt{3}\, 440 \times 0.8\right) = 0.66 \quad A$$

$$I_L = 400 / \left(\sqrt{3}\, 760 \times 0.8\right) = 0.38 \quad A$$

To calculate the motor's apparent power, we use the following formula:

$$P = S \cos\phi \quad \rightarrow \quad S = P / \cos\phi$$

$$S = 400 / 0.8 = 500 \quad VA$$

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For the motor's reactive power, we have:

$$Q = S\sin\phi \quad \cos\phi = 0.8 \quad \phi = \cos^{-1}0.8 = 36.87^0 \quad \rightarrow$$
$$Q = 500\sin 36.87 = 300 \quad VAr$$

The synchronous motor operates at the speed of the rotating magnetic field. To determine the total number of poles in the motor, we use the following equation:

$$N_s = 120f / p \rightarrow p = 120f / N_s =$$
$$120 \times 60 / 1800 = 4 \quad poles$$

EXAMPLE 2

Consider a three-phase synchronous generator with the following specifications: a voltage of 220 V, an active power of 0.4 kW, a power factor of 0.8, a frequency of 60 Hz, and a rotational speed of 1800 rpm. The field winding is supplied with 12 V DC and draws a current of 3 A. For the generator's output, determine the electric current, the apparent power, the reactive power, and the number of poles.

Solution

In a three-phase alternating current circuit, the line current can be determined from the total three-phase active power as follows:

$$P = \sqrt{3}V_L I_L \cos\phi \quad \rightarrow \quad I_L =$$
$$P / \left(\sqrt{3}V_L \cos\phi\right) =$$
$$400 / \left(\sqrt{3}220 \times 0.8\right) = 1.31 \quad A$$

The generator's apparent power is defined as follows:

$$S = \sqrt{3}V_L I_L = \sqrt{3}220 \times 1.31 = 499.18 \quad VA$$

The generator's reactive power is given by:

$$Q = \sqrt{3}V_L I_L \sin\phi \quad \cos\phi = 0.8$$
$$\phi = \cos^{-1}0.8 = 36.87^0$$
$$Q = \sqrt{3}220 \times 1.31\sin 36.87 = 299.51 \quad VAr$$

The synchronous generator operates at the speed of the rotating magnetic field. The total number of poles is determined by:

$$N_s = 120f / p \quad \rightarrow \quad p = 120f / N_s$$
$$= 120 \times 60 / 1800 = 4 \quad poles$$

EXAMPLE 3

Consider a three-phase squirrel-cage induction motor with 2.2 kW, 220/380 V, 3465 rpm, 60 Hz, efficiency of 81.5%, and a power factor of 0.84, where the starting current ratio $I_P / I_N = 7$. Determine the following: the input electrical power required by the motor from the power grid, the electrical current drawn by the motor at each voltage level, the apparent and reactive power of the motor, the starting current drawn by the motor from the power grid at each voltage level, the number of poles of the motor, and the motor's slip.

Solution

The following formula defines the motor efficiency and allows the calculation of

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the input electrical power required by the motor:

$$\eta = P_{out} / P_{in} \quad \rightarrow \quad P_{in} = P_{out} / \eta =$$
$$2200 / 0.815 = 2699.39 \quad W$$

For a three-phase alternating current circuit, the total three-phase active power is given by:

$$P = \sqrt{3} V_L I_L \cos\phi \quad \rightarrow \quad I_L = P / \left(\sqrt{3} V_L \cos\phi\right)$$

For a power grid operating at 220 V or 380 V, the motor requires a certain amount of current given by:

$$I_L = 2699.39 / \left(\sqrt{3} 220 \times 0.84\right) = 8.43 \quad A$$
$$I_L = 2699.39 / \left(\sqrt{3} 380 \times 0.84\right) = 4.88 A$$

The apparent power of the motor can be calculated based on the active power as follows:

$$P = S\cos\phi \quad \rightarrow \quad S = P / \cos\phi$$
$$S = 2699.39 / 0.84 = 3213.56 \quad VA$$

The reactive power is given by:

$$Q = S\sin\phi \quad \cos\phi = 0.84 \quad \phi = \cos^{-1} 0.84 = 32.86^0$$
$$Q = 3213.56 \sin 32.86 = 1743{,}64 \quad VAr$$

The relationship between the starting current and the nominal current is:

$$I_p / I_N = 7 \quad \rightarrow \quad I_p = 7 I_N$$
$$for \quad 220 \quad V: \quad I_p =$$
$$7 \times 8.43 = 59.01 \quad A$$
$$for \quad 380 \quad V: \quad I_p = 7 \times 4.88 = 34.16 \quad A$$

Note that the motor's current demand at startup is much higher than its operating current (59.01 > 8.43 and 34.16 > 4.88 A).

To determine the motor pole number, consider that the speed of the rotating magnetic field equals the motor shaft speed.

$$N_s = 120 f / p \quad p = 120 f / N_s$$
$$p = 120 \times 60 / 3465 = 2.08$$

The total number of poles is an even integer, indicating that the motor has 2 poles. We will use the number of poles to calculate the correct speed of the rotating magnetic field: N_s = 120 x 60 / 2 = 3600 rpm.

The slip of an induction motor is defined as follows:

$$s = 100\left(N_s - N\right) / N_s =$$
$$100\left(3600 - 3465\right) / 3600 = 3.75\%$$

EXAMPLE 4

The specifications for a three-phase AC induction motor with a squirrel-cage rotor are as follows: 0.75 CV, 440 V, 71.5% efficiency, 3410 rpm, 60 Hz, a starting current ratio I_p/I_n = 5.2, and a power factor of 0.82. Determine the electrical input power absorbed from the grid, the current drawn from the grid, the apparent power, the reactive power, the starting current, the number of poles, and the motor slip.

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Solution

The efficiency of the motor is given by: $\eta = P_{out} / P_{in}$

The active input power of the motor will be found as follows:

$$1CV = 736W \quad 0.75CV\frac{736W}{1CV} = 552 \quad W$$

$$P_{in} = P_{out} / \eta = 552 / 0.715 = 772.03W$$

In a three-phase AC circuit, the total three-phase active power can be used to determine the required current for the motor:

$$P = \sqrt{3}V_L I_L \cos\phi \quad \rightarrow I_L = P / \left(\sqrt{3}V_L \cos\phi\right) \rightarrow I_L$$

$$I_L = 772.03 / \left(\sqrt{3}440 \times 0.82\right) = 1.24A$$

The motor's apparent power is given by:

$$S = \sqrt{3}V_L I_L \quad S = \sqrt{3}440 \times 1.24 = 945.01 \quad VA$$

The motor reactive power is given by:

$$Q = \sqrt{3}\,V_L I_L \sin\phi \quad \cos\phi = 0.82$$

$$\phi = \cos^{-1} 0.82 = 34.92^0$$

$$Q = \sqrt{3}440 \times 1.24 \times \sin 34.92 = 540.95VAr$$

The starting current of the motor can be calculated as follows:

$$I_p / I_N = 5.2 \quad I_p = 5.2 I_N \quad I_p = 5.2 \times 1.24 = 6.45 \quad A$$

To find the number of motor poles, it's necessary to consider that the speed of the rotating magnetic field equals the motor shaft speed:

$$N_s = 120f / p \quad p = 120f / N_s \quad p = 120 \times 60 / 3410 = 2.11$$

The total number of poles is an even integer. The motor has 2 poles. Using the number of poles, we will calculate the correct value of the speed of the rotating magnetic field:

$$N_s = 120 \times 60 / 2 = 3600 \quad rpm$$

The slip of an induction motor is defined as follows:

$$s = 100\left(N_s - N\right) / N_s$$

$$= 100\left(3600 - 3410\right) / 3600 = 5.28\%$$

EXMPLE 5

Consider a three-phase alternating current induction motor with a squirrel-cage rotor, 60 Hz, 0.46/0.75 kW, 1740/3480 rpm, a starting current ratio I_p/I_n = 7.5, 220/220 V, 2.40/2.70 A, and a power factor of 0.68 / 0.88. Determine the electrical input active power required by the motor from the grid for each speed level, along with the motor's efficiency, apparent and reactive power, starting current, number of poles, and slip.

Solution

In a three-phase alternating current circuit, the total active input power is given by:

$$P = \sqrt{3}V_L I_L \cos\phi \quad for \quad 1740$$

$$rpm : P = \sqrt{3}220 \times 2.40 \times 0.68 =$$

$$621.88 \quad W$$

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$$for \quad 3480 \quad rpm: P = \sqrt{3}220 \times 2.70 \times 0.88$$
$$= 905.38 \quad W$$

The efficiency of the motor is represented by:

$$\eta = P_{out} / P_{in} \quad for \quad 1740 \quad rpm: \eta = 460 / 621.88 = 73.97\%$$
$$for \quad 3480 \quad rpm: \eta = 750 / 905.38 = 82.84\%$$

The following equation describes the apparent power of the motor:

$$S = P / \cos\phi \quad for \quad 1740 \quad rpm: S = 621.88 / 0.68 = 914.53 \quad VA$$
$$for \quad 3480 \quad rpm: S = 905.38 / 0.88 = 1028.84 \quad VA$$

The reactive power of the motor is given by: $Q = S\sin\phi$

$$for \quad 1740 \quad rpm: \quad \cos\phi = 0.68 \quad \phi = \cos^{-1} 0.68 = 47.16^0$$
$$Q = 914.53 \sin 47.16 = 670.58 VAr$$
$$for \quad 3480 \quad rpm: \quad \cos\phi = 0.88 \quad \phi = \cos^{-1} 0.88 = 28.36^0$$
$$Q = 1028.84 \sin 28.36 = 488.71 VAr$$

The starting current of the motor is defined as follows:

$$I_p / I_N = 7.5 \quad \rightarrow \quad I_p = 7.5 I_N \quad for \quad 1740$$
$$rpm: I_p = 7.5 \times 2.40 = 18.00 \quad A$$
$$for \quad 3480 \quad rpm: I_p = 7.5 \times 2.70 = 20.25 \quad A$$

To find the number of motor poles, it's necessary to consider that the speed of the rotating magnetic field equals the motor shaft speed:

$$N_s = 120 f / p \quad p = 120 f / N_s \quad for \quad 1740$$
$$rpm: p = 120 \times 60 / 1740 = 4.14$$

$$for \quad 3480 \quad rpm: p = 120 \times 60 / 3480 = 2.07$$

The total number of poles in the motor is an even integer. Therefore, the motor has 4 poles at 1740 rpm and 2 poles at 3480 rpm. Using the number of poles, we can calculate the correct value of the rotating magnetic field speed:

$$N_s = 120 \times 60 / 4 = 1800 \quad rpm$$
$$for\ a\ shaft\ with \quad 1740 \quad rpm$$

$$N_s = 120 \times 60 / 2 = 3600 \quad rpm$$
$$for\ a\ shaft\ with \quad 3480 \quad rpm$$

The slip of the motor is defined as follows:

$$s = 100(N_s - N) / N_s \quad for\ a\ shaft\ with$$
$$1740 \quad rpm: s = 100(1800 - 1740)/1800 = 3.33\%$$

$$for\ a\ shaft\ with \quad 3480 \quad rpm: s =$$
$$100(3600 - 3480)/3600 = 3.33\%$$

EXAMPLE 6

Consider a three-phase squirrel-cage induction motor with the following specifications: 2.2 kW, 3440 rpm, 380 V, 60 Hz, power factor 0.85 lagging, starting current ratio I_p/I_n 7.8, and efficiency 85.1%. A capacitor bank is connected in a delta configuration across the supply terminals to improve the power factor to 0.95 lagging. Figure 1 illustrates the circuit configurations before and after the power factor improvement.

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Determine the capacitance of the capacitors used in each phase of the delta bank.

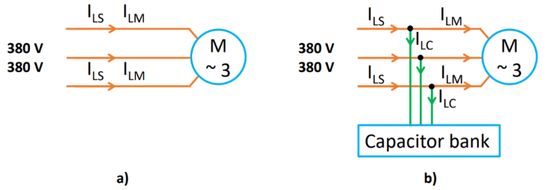

Figure 1 – Power factor improvement: a) electrical circuit before improvement, b) electrical circuit after improvement.

Source: Own authorship.

Solution

Before improving the power factor for the three-phase motor, we observed the following:

$$\eta = P_{out} / P_{in} \quad P_{in} = P_{out} / \eta = 2200 / 0.851 = 2585.19$$

$$P_{inm} = \sqrt{3} V_{Lm} I_{Lm} \cos\phi_m = 2585.19$$

$$\sqrt{3} 380 I_{Lm} 0.85 = 2585.19 \quad I_{Lm} = 2585.19 / \left(\sqrt{3} 380 \times 0.85\right) = 4.62A \quad \cos\phi_m = 0.85$$

$$\phi_m = ar\cos 0.85 = 31.79^0$$

$$Q_m = \sqrt{3} V_{Lm} I_{Lm} sin\phi_m$$

$$Q_m = \sqrt{3} 380 \times 4.62 \times \sin 31.79^0$$

$$Q_m = 1601.91 \quad VAr$$

$$S_m = \sqrt{3} V_{Lm} I_{Lm} \quad S_m = \sqrt{3} 380 \times 4.62$$

$$S_m = 3040.79 \quad VA$$

The motor draws an electric current of 4.62 A from the power supply. After improving the power factor for the system, we have the following:

$$\cos\phi_s = 0.95 \quad \phi_s = ar\cos 0.95 = 18.19^0$$

$$P_s = \sqrt{3} V_{Ls} I_{Ls} \cos\phi_s$$

$$2585.19 = \sqrt{3} 380 \times I_{Ls} \times 0.95 \quad W$$

$$I_{Ls} = 2585.19 / \left(\sqrt{3} 380 \times 0.95\right)$$

$$I_{Ls} = 4.13A \quad Q_s = \sqrt{3} V_{Ls} I_{Ls} sin\phi_s$$

$$Q_s = \sqrt{3} 380 \times 4.13 \times \sin 18.19^0$$

$$Q_s = 848.56 \quad VAr \quad S_s = \sqrt{3} V_{Ls} I_{Ls}$$

$$S_s = \sqrt{3} 380 \times 4.13 \quad S_s = 2718.28 \quad VA$$

The power of 2585.19 W remains constant because the capacitor added to the circuit does not dissipate power, as demonstrated in the following steps. The capacitor must provide reactive power, which, when combined with the motor's reactive power, contributes to the system´s overall reactive power. For the capacitor, we have the following:

$$1601.91 + Q_C = 848.56 \quad Q_C = 848.56 - 1601.91$$

$$Q_C = -753.35 \quad VAr$$

$$Z_c = -j / (2\pi f C) = \left(1 / (2\pi f C)\right) \measuredangle -90^0$$

$$\Omega \quad \phi_c = -90^0 \quad Q_c = \sqrt{3} V_{Lc} I_{Lc} sin\phi_c$$

$$-753.35 = \sqrt{3} 380 I_{Lc} \sin(-90)$$

$$I_{Lc} = -753.35 / \left(\sqrt{3} 380 \times \sin(-90)\right) = 1.14 \quad A$$

$$P_c = \sqrt{3} V_{Lc} I_{Lc} \cos\phi_c \quad P_c = \sqrt{3} 380 \times 1.14 \times \cos(-90) = 0$$

$$W \quad S_c = \sqrt{3} V_{Lc} I_{Lc} \quad S_c = \sqrt{3} 380 \times 1.14$$

$$S_c = 750.32 \quad VA \quad delta \quad connection: \quad V_L = V_f \quad I_L = \sqrt{3} I_f$$

$$Z_c = V_{fc} / I_{fc} \quad 1/(2\pi f C) = 380/(1.14/\sqrt{3})$$

$$= 577.35 \quad \Omega \quad 1/(2\pi 60 C) = 577.35$$

$$C = 1/(2\pi 60 \times 577.35) \quad C = 4.59 \quad \mu F$$

Before the power factor improvement, the system required a current of 4.62 A. After increasing the power factor, this current decreased to 4.13 A, while the motor's current and power remained unchanged. As a result, the 380 V generating plant supplying the system can save costs by using smaller gauge conductors in the electrical grid.

EXAMPLE 7

In the previous exercise, each phase of the delta connection had three identical 127 V capacitors connected in series in the capacitor bank. Calculate the capacitance of each capacitor.

Solution

Three capacitors connected in series have the following equivalent capacitance:

$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 \quad 1/C_{eq} = 1/$$
$$C + 1/C + 1/C = 3/C \quad C = 3C_{eq} = 3 \times 4.59$$
$$C = 13.77 \quad \mu F$$

Each capacitor must have 13.77 microfarads.

EXAMPLE 8

Consider a single-phase induction motor with a 1/3 CV, 3520 rpm, and a 110/220 V voltage rating, drawing 6.8/3.4 A. The starting current ratio I_p/I_n is 5.3 A, and the frequency is 60 Hz. Calculate the starting current for each voltage level.

Solution

The starting current ratio is:

$$I_p / I_N = 5.3 \quad \rightarrow \quad I_p = 5.3 I_N \quad for$$
$$110 \quad V: I_p = 5.3 \times 6.8 = 36.04 \quad A$$
$$for \quad 220 \quad V: I_p = 5.3 \times 3.4 = 18.02 \quad A$$

EXAMPLE 9

Consider an AC single-phase motor, 0.37 kW, 3465 rpm, 110/220 V, 7.00/3.50 A, 60 Hz, power factor = 0.78, the starting current ratio I_p/I_N = 6. Calculate the following for each voltage level: the input active power, the efficiency, the apparent power, the reactive power, and the starting current.

Solution

In a single-phase AC circuit, the active power is defined as:

$$P = V_{ef} I_{ef} \cos\phi \quad for \quad 110$$
$$V: P = 110 \times 7 \times 0.78 = 600.6 \quad W$$
$$for \quad 220 \quad V: P = 220 \times 3.5 \times 0.78 = 600.6 \quad W$$

The efficiency of the motor is expressed as follows: $\eta = P_{out} / P_{in}$

$$for \quad 110 \quad V: \eta = 370 / 600.6 = 61.60\%$$
$$for \quad 220 \quad V: \eta = 370 / 600.6 = 61.60\%$$

The apparent power of the motor is given by: $S = V_{ef} I_{ef}$

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$$for \quad 110 \quad V: S = 110 \times 7 = 770 \quad VA$$

$$for \quad 220 \quad V: S = 220 \times 3.5 = 770 \quad VA$$

Reactive power is defined as follows: *Q* = *S*sinø

$$\cos\phi = 0.78 \quad \phi = \cos^{-1} 0.78 = 38.74^0$$

$$for \quad 110 \quad \mathrm{V}: Q = 770 \sin 38.74 = 481.86 \quad VAr$$

$$for \quad 220 \quad \mathrm{V}: Q = 770 \sin 38.74 = 481.86 \quad VAr$$

The starting current ratio is given by:

$$I_p / I_N = 6 \quad \rightarrow \quad I_p = 6I_N \quad for$$

$$110 \quad V: I_p = 6 \times 7 = 42.00 \quad A$$

$$for \quad 220 \quad V: I_p = 6 \times 3.50 = 21.00 \quad A$$

EXAMPLE 10

Consider an AC single-phase motor, 0.37 kW, 3465 rpm, 220 V, 3.50 A, 60 Hz, with a lagging power factor of 0.78, and a starting current ratio I_p/I_n = 6. Determine the capacitance of the capacitor connected in parallel with the motor to achieve a lagging power factor of 0.9. Figure 2 shows the circuit configurations before and after the power factor correction.

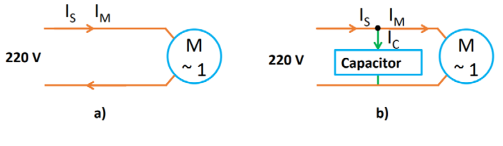

Figure 2 – Power factor improvement: a) electrical circuit before improvement, b) electrical circuit after improvement.

Source: Own authorship.

Solution

Before improving the power factor for the single-phase motor, we observed the following:

$$P_m = V_{efm} I_{efm} \cos\phi_m \quad P_m = 220 \times 3.50 \times 0.78 = 600.6$$

$$W \quad \cos\phi_m = 0.78 \quad \phi_m = ar\cos 0.78 = 38.74^0$$

$$Q_m = V_{efm} I_{efm} sin\phi_m \quad Q_m = 220 \times 3.50 \times \sin 38.74^0$$

$$Q_m = 481.86 \quad VAr$$

$$S_m = V_{efm} I_{efm} \quad S_m = 220 \times 3.50 \quad S_m = 770 \quad VA$$

The motor draws an electric current of 3.50 A from the power supply. After improving the power factor for the system, we have the following:

$$\cos\phi_s = 0.9 \quad \phi_s = ar\cos 0.9 = 25.84^0$$

$$P_s = V_{efs} I_{efs} \cos\phi_s \quad P_s = 220 \times I_s \times 0.9 = 600.6 \quad W$$

$$I_s = 600.6 / (220 \times 0.9) \quad I_s = 3.03$$

$$A \quad Q_s = V_{efs} I_{efs} sin\phi_s \quad Q_s = 220 \times 3.03 \times \sin 25.84^0$$

$$Q_s = 290.54 \quad VAr \quad S_s = V_{efs} I_{efs}$$

$$S_s = 220 \times 3.03 \quad S_s = 666.6 \quad VA$$

The power of 600.6 W remains constant because the capacitor added to the circuit does not dissipate power, as demonstrated in the following steps. The capacitor must provide reactive power, which, when combined with the motor's reactive power, contributes to the system´s overall reactive power. For the capacitor, we have the following:

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$481.86 + Q_C = 290.54 \quad Q_C = 290.54 - 481.86$

$Q_C = -191.32 \quad VAr \quad Q_c = V_{efc} I_{efc} sin\phi_c$

$Q_c = 220 I_c sin\phi_c \quad Z_c = -j/(2\pi fC) =$

$1/(2\pi fC)\measuredangle -90^0 \quad \Omega \quad \phi_c = -90^0$

$Q_c = 220 I_c sin\phi_c \quad -191.32 = 220 I_c \sin(-90)$

$I_c = -191.32/(220 \times \sin(-90)) = 0.87 \quad A$

$P_c = V_{efc} I_{efc} \cos\phi_c \quad P_c = 220 \times 0.87 \times \cos(-90) =$

$0 \quad W \quad S_c = V_{efc} I_{efc} \quad S_c = 220 \times 0.87 \quad S_c = 191.40 VA$

$Z_c = V_c / I_c \quad 1/(2\pi fC) = 220/0.87 \quad 1/(2\pi 60C)$

$= 220/0.87 \quad C = 0.87/(2\pi 60 \times 220) \quad C = 10.49 \mu F$

EXAMPLE 11

Consider an AC single-phase induction motor with a squirrel-cage rotor, 0.37 kW, 3400 rpm, 220 V, 2.60 A, 60 Hz, power factor 0.94, and a starting current ratio I_p/I_n = 3.5. Determine the input active power of the motor, the efficiency, the apparent and the reactive power, and the starting current of the motor.

Solution

In a single-phase AC circuit, the active power is defined as:

$$P = V_{ef} I_{ef} \cos\phi \quad P = 220 \times 2.60 \times 0.94 = 537.68 \quad W$$

The efficiency of the motor is expressed as follows:

$$\eta = P_{out} / P_{in} \quad \eta = 370/537.68 = 68.81\%$$

The apparent power of the motor is given by:

$$S = V_{ef} I_{ef} \quad S = 220 \times 2.60 = 572 \quad VA$$

Reactive power is defined as follows:

$$Q = S\sin\phi \quad \cos\phi = 0.94 \quad \phi = \cos^{-1} 0.94 = 19.95^0$$

$$Q = 572 \sin 19.95 = 195.17 \quad VAr$$

The starting current ratio is given by:

$$I_p / I_N = 3.5 \quad I_p = 3.5 I_N$$

$$I_p = 3.5 \times 2.60 = 9.1 \quad A$$

The motor consumes more current during startup than during operation (9.1 > 2.60 A).

CONCLUDING REMARKS

This chapter discussed the solving exercises related to synchronous motors, synchronous generators, and both single--phase and three-phase induction motors. Step-by-step solved exercises are essential for effective learning. These examples offer a clear and practical way to understand how theory is applied in real scenarios, providing several benefits: they help students identify and address gaps in their understanding, serve as models for structuring solutions, highlight common mistakes, provide a reliable foundation for future learning, and act as a silent teacher.

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